Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **9FM0/3C**

Further Mathematics

Advanced

PAPER 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over

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P 6 6 8 0 0 A 0 1 2 8

Pearson

1. A van of mass 900 kg is moving along a straight horizontal road.

At the instant when the speed of the van is $v\,\text{m s}^{-1}$, the resistance to the motion of the van is modelled as a force of magnitude $(500 + 7v)$ N.

When the engine of the van is working at a constant rate of 18 kW, the van is moving along the road at a constant speed $V\,\text{m s}^{-1}$

(a) Find the value of V.

(5)

Later on, the van is moving up a straight road that is inclined to the horizontal at an angle θ, where $\sin\theta = \dfrac{1}{21}$

At the instant when the speed of the van is $v\,\text{m s}^{-1}$, the resistance to the motion of the van from non-gravitational forces is modelled as a force of magnitude $(500 + 7v)$ N.

The engine of the van is again working at a constant rate of 18 kW.

(b) Find the acceleration of the van at the instant when $v = 15$

(4)

(a) Let's illustrate the above information on a detailed force diagram:

...label the variable resistance, the weight and the POWER rearranged:

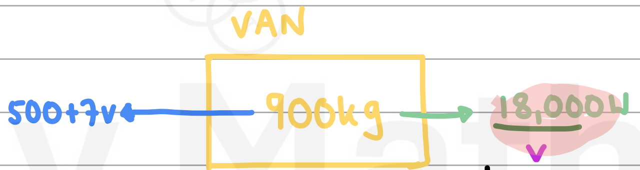
know $v = V$

formula: $P = Fv$
Power in Watts, Force in Newtons, velocity in ms^{-1}

$\Rightarrow F = \dfrac{P}{v}$

but here $P = 18\text{kW} \xrightarrow[\text{convert to Watts}]{\times 1000} 18{,}000\text{W}$

and $v = V$



NOTE: could've calculated this power as separate line of working but much more efficient in the exam to straight away write in the force from the formula rearranged

know from Yr 1 Mechanics Chp 8 that the fact that the van is moving 'at constant speed' means its acceleration is zero (object is at non-stationary equilibrium)

so right components of force diagram = left

$R(\leftrightarrow)$: $500 + 7V = \dfrac{18{,}000}{V}$

$\times V$ $\times V$

solve for 'V'

P 6 6 8 0 0 A 0 2 2 8

Question 1 continued

$500v + 7v^2 = 18{,}000$

$\Leftrightarrow 7v^2 + 500v - 18{,}000 = 0$

solve using calc eqtn solver or using quadratic formula:

$$V = \frac{-500 \pm \sqrt{(500)^2 - 4(7)(-18{,}000)}}{2(7)}$$

$$= \frac{-500 \pm \sqrt{754{,}000}}{14}$$

…+ve: $V = 26.3094\ldots$ …-ve: $v = -97.73\ldots$

but assume van moving with +ve speed (scalar)

$\boxed{\therefore\ v = 26\,ms^{-1}\ (2\ s.f)}$

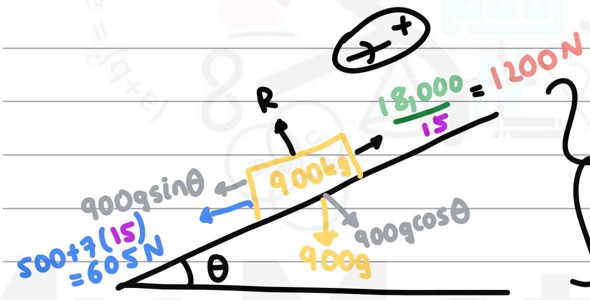
(b) let's look at the forces again - redrawing part (a) diagram but on an inclined plane - label variable resistance, weight, the POWER rearranged

$\underbrace{\text{variable resistance}}_{\text{where } v = 15\,ms^{-1}}$

formula: $P = Fv$

$F = \frac{P}{v}$

where $P = 18{,}000W$ and $v = 15\,ms^{-1}$



need the acceleration - from Mechanics Yr 1 Chp 8

know to resolve parallel to the plane and input into Newton's Second Law

formula: $\Sigma F_x = ma$

$R(\nearrow)$: $1200 - 605 - 900g\sin\theta = 900a$

given $\sin\theta = 1/21$ in the question:

$1200 - 605 - 900(9.8)(1/21) = 900a$

$\Rightarrow 595 - 420 = 900a$

$175 = 900a$

$\div 900 \qquad \div 900$

$\Rightarrow a = \frac{175}{900} = \frac{7}{36} = 0.1944\ldots$

or $\boxed{0.19\,ms^{-2}\ (2\,d.p)}$

Question 1 continued

Question 1 continued

(Total for Question 1 is 9 marks)

2. Two particles, A and B, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.

Particle A has mass $5m$ and particle B has mass $3m$.

The coefficient of restitution between A and B is e, where $e > 0$

Immediately **after** the collision the speed of A is v and the speed of B is $2v$.

Given that A and B are moving in the same direction after the collision,

(a) find the set of possible values of e.

(8)

Given also that the kinetic energy of A immediately after the collision is 16% of the kinetic energy of A immediately before the collision,

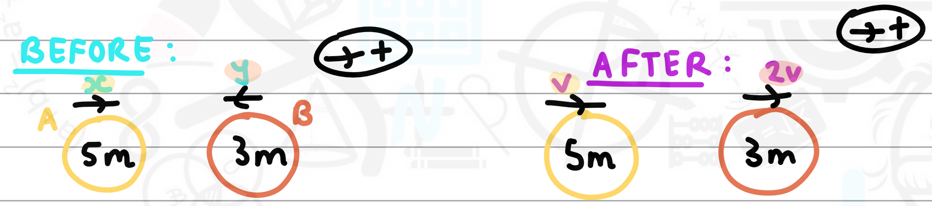
(b) find

(i) the value of e,

(ii) the magnitude of the impulse received by A in the collision, giving your answer in terms of m and v.

(6)

(a) illustrating this elastic collisions in 1D diagrammatically - label the respective speeds, direction of motion, etc.



following the usual procedure for elastic collisions in 1D - notice both speeds before are unknown ∴ can't just stop at using PCLM - need to do NEL (impact law) as well

...first PCLM - means the total momentum before the collision equals the total momentum after :

formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

sub into above

$5m(x) + 3m(-y) = 5m(v) + 3m(2v)$

expand brackets

$5\cancel{m}x - 3\cancel{m}y = 11\cancel{m}v$

cancel m's

$5x - 3y = 11v$ ①

...next, NEL - i.e the formula for coefficient of restitution :

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Question 2 continued

formula: $e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{2v - v}{x - (-y)}$

$\Rightarrow e = \frac{v}{x+y}$

$\times(x+y) \qquad \times(x+y)$

$e(x+y) = v$

expand

$ex + ey = v$ ②

solving ① and ② simultaneously - first eliminate 'x':

① × e − ② × 5

$5ex - 3ey = 11ev$

$-\ 5ex + 5ey = 5v$

$-8ey = 11ev - 5v$

$\div -8e \qquad \div -8e$ and factorise 'v' out

$y = -\frac{v}{8e}(11e - 5)$

or factor -ve into the bracket

$\boxed{\Rightarrow y = \frac{v}{8e}(5 - 11e)}$

...next, eliminate 'y':

① × e + ② × 3

$5ex - 3ey = 11ev$

$+\ 3ex + 3ey = 3v$

$8ex = 11ev + 3v$

$\div 8e \qquad \div 8e$ and factorise 'v' out:

$\boxed{x = \frac{v}{8e}(11e + 3)}$

finally, the only source of inequality given in the question is that $e > 0$, which implies that $u_A > 0$ is moving in the +ve direction, and u_B in the -ve. However, we've already assumed that in the diagram, so for us, $u_B > 0$

$\frac{v}{8e}(5 - 11e) > 0$

$\Rightarrow 5 - 11e > 0$

$\Rightarrow 11e < 5$

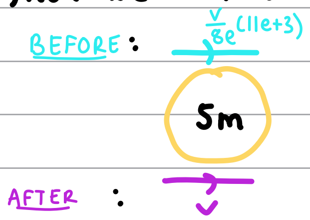
$\div 11 \qquad \div 11$

$\Rightarrow \boxed{e < 5/11}$

Question 2 continued

and remember $0 \le e \le 1$, so the full range for 'e' must be $\boxed{0 < e < 5/11}$

(b)(i) now we want to just focus on A:



we are given that the $E_k \text{initial} \times \frac{16}{100} = E_k \text{final}$, so need to find both E_k initial and E_k final and sub into above equation

...first, find E_k initial:

formula: $\frac{1}{2}m(u_A)^2$

$= \frac{1}{2}(5m)\left(\frac{v}{8e}(11e+3)\right)^2$

$= \frac{5}{2}m\left(\frac{v^2}{64e^2}(11e+3)^2\right)$

$$E_k \text{initial} = \frac{5mv^2}{128e^2}(11e+3)^2$$

...next, find E_k final:

formula: $\frac{1}{2}m(v_A)^2$

$= \frac{1}{2}(5m)(v)^2$

$= \frac{5mv^2}{2}$

$$\Rightarrow E_k \text{final} = \frac{5mv^2}{2}$$

subbing into the equation:

$$\frac{16}{100}\left(\frac{5\cancel{mv^2}}{128e^2}\right)(11e+3)^2 = \frac{5}{2}\cancel{mv^2}$$

cancel the mv^2

$$\frac{1}{160e^2}(11e+3)^2 = \frac{5}{2}$$

expand the double brackets

$$\frac{1}{160e^2}(121e^2+66e+9) = \frac{5}{2}$$

$\times 160e^2$ $\qquad$ $\times 160e^2$

$$121e^2+66e+9 = 400e^2$$

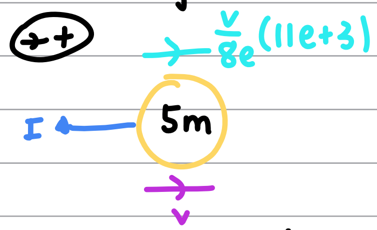
$$279e^2-66e-9=0$$

calc equtn solver

$$\Rightarrow e = 1/3 \text{ or } \underbrace{-3/31}_{\text{reject as } 0 \le e \le 1}$$

$\boxed{\therefore e = 1/3}$

(ii) now to find impulse on A, notice that it acts to the LEFT (due to particle B moving to the left before colliding with A) - illustrating on a detailed diagram:



...using Impulse-momentum formula:

formula: $I = m(v-u)$

also know $e = 1/3$, so sub that into u_A

$$u_A = \frac{v}{8(1/3)}\left(11(1/3)+3\right)$$

$$\Rightarrow u_A = \frac{3v}{\cancel{8}_2}\left(\frac{\cancel{20}^5}{\cancel{3}}\right)$$

$$= \frac{5v}{2}$$

and know $v_A = v$

subbing velocities into formula:

$$-I = 5m\left(v - \frac{5v}{2}\right)$$

$$-I = 5m\left(-3/2\, v\right)$$

$$-I = -\frac{15m}{2}v$$

$$\therefore \boxed{I = \frac{15mv}{2}\ \text{Ns}}$$

(Total for Question 2 is 14 marks)

3. [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A smooth uniform sphere P has mass 0.3 kg. Another smooth uniform sphere Q, with the same radius as P, has mass 0.5 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision the velocity of P is $(u\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$, where u is a positive constant, and the velocity of Q is $(-4\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$

At the instant when the spheres collide, the line joining their centres is parallel to **i**.

The coefficient of restitution between P and Q is $\frac{3}{5}$

As a result of the collision, the direction of motion of P is deflected through an angle of 90° and the direction of motion of Q is deflected through an angle of $\alpha°$

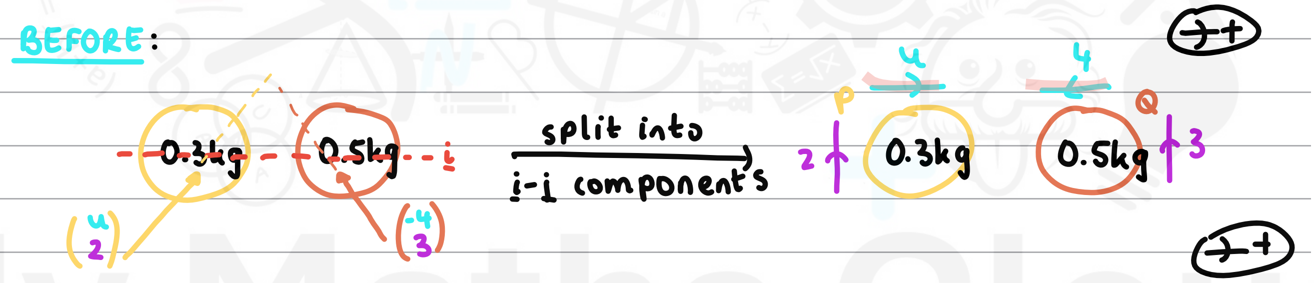
(a) Find the value of u **(8)**

(b) Find the value of α **(5)**

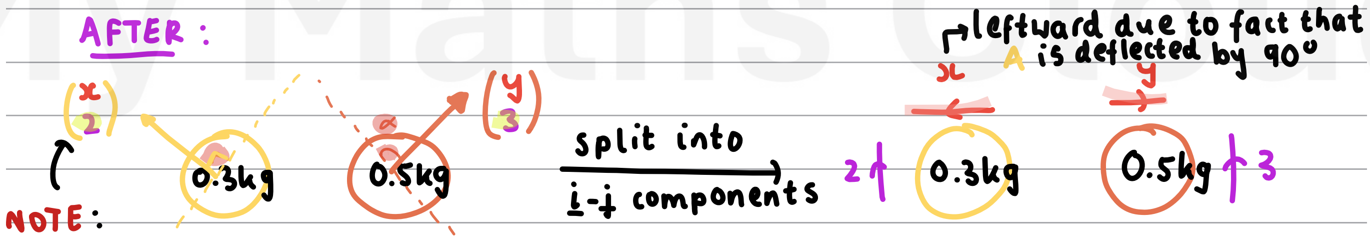
(c) State how you have used the fact that P and Q have equal radii. **(1)**

(a) notice we have an 'oblique collision between two spheres' question - first illustrating the collision with a diagram:

BEFORE:



AFTER:



NOTE: perpendicular components of velocity always remain the same

...parallel components: notice how speeds after are unknown, so can't stop at just using PCLM - need NEL (impact law) as well:

...first, PCLM, i.e total momentum before the collision equals total momentum after the collision

P 6 6 8 0 0 A 0 1 0 2 8

Question 3 continued

formula: $m_P u_P + m_Q u_Q = m_P v_P + m_Q v_Q$

$0.3(u) + 0.5(-4) = 0.3(-x) + 0.5(y)$

$\Rightarrow 0.5y - 0.3x = 0.3u - 2$ ①

...next, NEL:

formula: $e = \dfrac{\text{speed of separation}}{\text{speed of approach}}$

$\dfrac{3}{5} = \dfrac{y-(-x)}{u-(-4)}$

$\Rightarrow 0.6(u+4) = y + x$

$\therefore x + y = 0.6u + 2.4$ ②

Now use the fact that A has been deflected by 90° – 2 ways to do this:

WAY 1: using scalar dot product fact for perpendicular components

formula: $\underline{a} \cdot \underline{b} = 0$

so $\begin{pmatrix} u \\ 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ 2 \end{pmatrix} = 0$

evaluate scalar dot product

$ux + 4 = 0$

$ux = -4$

$\div u \quad \div u$

$x = -4/u$

but already considered on diagram that x is going left so $x > 0$

$\therefore x = 4/u$

WAY 2: looking at P geometrically and using gradients

formula: $m_1 \times m_2 = -1$

$\dfrac{2}{u} \times \dfrac{2}{x} = \dfrac{4}{ux} = -1$

solve for 'x'

$\Rightarrow 4 = -ux$

$\div -u \quad \div -u$

$\Rightarrow x = -\dfrac{4}{u}$

but already considered on diagram that x is going left so $x > 0$

$\therefore x = 4/u$

subbing this 'x' into ① and ②

into ① : $0.5y - 0.3\left(\dfrac{4}{u}\right) = 0.3u - 2$

expand brackets out

$0.5y - \dfrac{1.2}{u} = 0.3u - 2$ ③

into ② : $\left(\dfrac{4}{u}\right) + y = 0.6u + 2.4$ ④

now need to solve ③ and ④ simultaneously – elim. 'y' for 'u'

Question 3 continued

② × 0.5 − ①

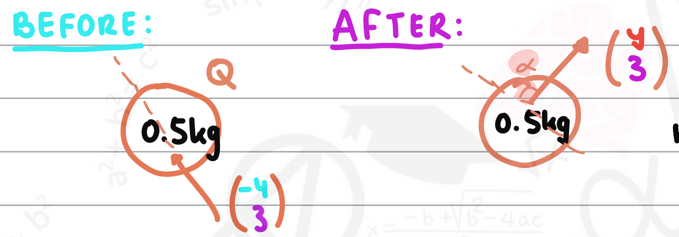
$$0.5y + \frac{2}{u} = 0.3u + 1.2$$
$$-\ \ 0.5y - \frac{1.2}{u} = 0.3u - 2$$

$$\frac{3.2}{u} = 3.2$$

solve for 'u'

$$\boxed{u = \frac{3.2}{3.2} = 1}$$

(b) 'α' relates to the angle of deflection of Q, hence focusing on the before and after of Q:



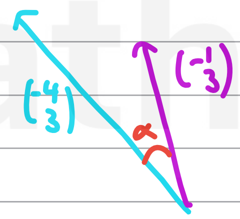
hence need to find 'y'

subbing in u=1 to any of ① or ② for 'y'

eg. into ① $0.5y - \frac{1.2}{1} = 0.3(1) - 2$

$$\Rightarrow 0.5y = -0.5$$
$$\div 0.5 \qquad \div 0.5$$
$$\Rightarrow y = -1$$

Now want the angle of deflection between velocity before and velocity after of Q



...two main ways to do this:

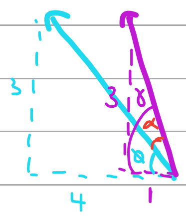
WAY 1: using angle between two vector lines formula

formula: $\cos\theta = \dfrac{\underline{u}_Q \cdot \underline{v}_Q}{|\underline{u}_Q||\underline{v}_Q|}$ ← scalar product ← product of magnitudes

Subbing in our values:

$$\cos\alpha = \frac{\begin{pmatrix}-4\\3\end{pmatrix} \cdot \begin{pmatrix}-1\\3\end{pmatrix}}{\sqrt{(-4)^2+(3)^2}\sqrt{(-1)^2+(3)^2}}$$

WAY 2: non-scalar product way, i.e geometrically with triangles and trig



See how 'α' comes from angle of blue triangle, call it 'θ', subtracted from angle of purple triangle, call it γ

$$\Rightarrow \alpha = \gamma - \theta$$

Question 3 continued

$\Rightarrow \cos\alpha = \frac{13}{5\sqrt{10}}$

$\alpha = \cos^{-1}\left(\frac{13}{5\sqrt{10}}\right)$

$= 34.6951..$

$\Rightarrow \boxed{\alpha = 34.7° \, (3 \, s.f)}$

$\alpha = \tan^{-1}(3/1) - \tan^{-1}(3/4)$

$= 34.6951...$

$\boxed{= 34.7° \, (3 \, s.f)}$

(Total for Question 3 is 14 marks)

4. A particle P has mass 0.5 kg. It is moving in the xy plane with velocity $8\mathbf{i}\,\mathrm{m\,s^{-1}}$ when it receives an impulse $\lambda(-\mathbf{i}+\mathbf{j})\,\mathrm{N\,s}$, where λ is a positive constant.

The angle between the direction of motion of P immediately before receiving the impulse and the direction of motion of P immediately after receiving the impulse is $\theta°$

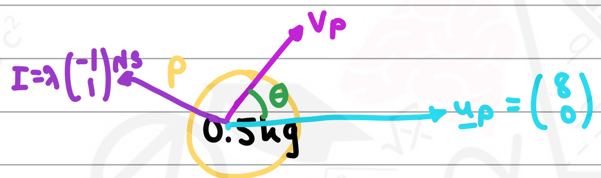
Immediately after receiving the impulse, P is moving with speed $4\sqrt{10}\,\mathrm{m\,s^{-1}}$

Find (i) the value of λ

(ii) the value of θ

(8)

notice we're dealing with momentum and impulse as VECTORS, so have to illustrate P's motion with a clear diagram indicating direction of motion, initial velocity, impulse, final velocity, and angle of deflection



(i) with above information, can use the vector formula for Impulse-momentum:

formula: $I = m(\underline{v}-\underline{u})$

sub in information

$$\begin{pmatrix}-\lambda\\\lambda\end{pmatrix} = 0.5\left(\underline{v_p} - \begin{pmatrix}8\\0\end{pmatrix}\right)$$

expand into bracket

$$\begin{pmatrix}-\lambda\\\lambda\end{pmatrix} = 0.5\underline{v_p} - \begin{pmatrix}4\\0\end{pmatrix}$$

$$\Rightarrow 0.5\underline{v_p} = \begin{pmatrix}-\lambda+4\\\lambda\end{pmatrix}$$

$\div 0.5$ $\div 0.5$

$$\underline{v_p} = \begin{pmatrix}-2\lambda+8\\2\lambda\end{pmatrix}$$

and given that speed after has a magnitude of $4\sqrt{10}\,\mathrm{ms^{-1}}$, we can use Pythagoras' on v_p

$$4\sqrt{10} = \sqrt{(-2\lambda+8)^2+(2\lambda)^2}$$

square both sides

$$160 = 4\lambda^2 - 32\lambda + 64 + 4\lambda^2$$

$$\Rightarrow 8\lambda^2 - 32\lambda - 96 = 0$$

$\div 8$ $\div 8$

$$\lambda^2 - 4\lambda - 12 = 0$$

↳ this is easily factorisable

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Question 4 continued

$(\lambda-6)(\lambda+2)=0$

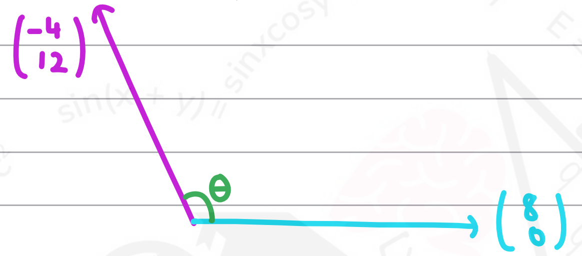
$\Rightarrow \lambda=6$ or $\lambda=-2$

↳ reject, as want λ to be a +ve constant

$\boxed{\therefore \lambda=6}$

(ii) for angle of deflection need the $\underline{v}_p$ vector - subbing in $\lambda=6$ into the vector for v_p = $\begin{pmatrix}-2(6)+8\\2(6)\end{pmatrix} = \begin{pmatrix}-4\\12\end{pmatrix}$

so now looking at the motion of P:



two main ways to find this angle of deflection:

WAY 1: using angle between two lines formula

$\cos\theta = \dfrac{u_p \cdot v_p}{|u_p||v_p|}$ ← scalar product ← product of magnitudes

$\cos\theta = \dfrac{\begin{pmatrix}8\\0\end{pmatrix}\cdot\begin{pmatrix}-4\\12\end{pmatrix}}{\sqrt{(8)^2+(0)^2}\sqrt{(-4)^2+(12)^2}}$

$\cos\theta = \dfrac{-32}{32\sqrt{10}} \Rightarrow \cos\theta = -\dfrac{1}{\sqrt{10}}$

take inverse cos of both sides

$\theta = 108.434...$

$\boxed{\theta = 108° (3 s.f)}$

WAY 2: non-formula method - using vector triangle and trig

see that because $\begin{pmatrix}8\\0\end{pmatrix}$ is a straight line, we can use the angles on a straight line property

$\theta = 180° - \gamma$

$= 180° - \tan^{-1}\left(\frac{12}{4}\right)$

$= 180° - \tan^{-1}(3)$

$= 108.4349...$

$\boxed{= 108° (3 s.f)}$

(Total for Question 4 is 8 marks)

5.

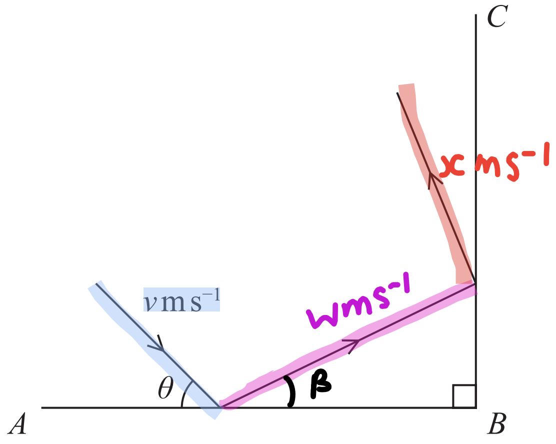


Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where AB and BC represent fixed vertical walls, with AB perpendicular to BC.

A small ball is projected along the floor towards the wall AB. Immediately before hitting the wall AB the ball is moving with speed $v\text{ m s}^{-1}$ at an angle θ to AB.

The ball hits the wall AB and then hits the wall BC.

The coefficient of restitution between the ball and the wall AB is $\frac{1}{3}$

The coefficient of restitution between the ball and the wall BC is e.

The floor and the walls are modelled as being smooth.

The ball is modelled as a particle.

The ball loses half of its kinetic energy in the impact with the wall AB.

(a) Find the exact value of $\cos\theta$.

(5)

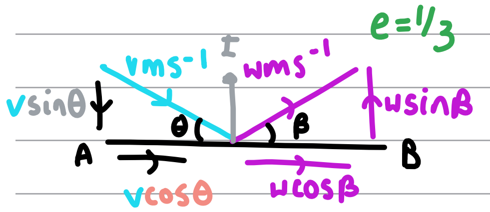
The ball loses half of its remaining kinetic energy in the impact with the wall BC.

(b) Find the exact value of e.

(5)

notice we are dealing with successive oblique collisions with two fixed surfaces - first considering the first collision with AB, as given the most information about it

FIRST COLLISION



...perpendicular:

remembering how when a particle collides obliquely with a fixed surface, the IMPULSE acts perpendicular to the plane of impact ∴ only perp. components of the initial 'v' ms⁻¹ change - here, NEL rearranged applies

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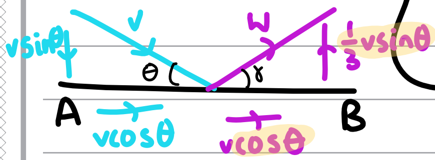
Question 5 continued

$w\sin\beta = e v \sin\theta$

$w\sin\beta = \frac{1}{3} v\sin\theta$ ①

.. putting these on diagram:



... parallel :

don't change as no impulse that acts parallel to wall AB, hence :

$w\cos\beta = v\cos\theta$ ②

now let's use the fact that the ball loses half of its E_k in the impact with the wall AB (so $\frac{1}{2} \times E_{k\,initial} = E_{k\,final}$)

... first finding $E_{k\,initial}$:

formula: $\frac{1}{2}mv^2$

$\therefore E_{k\,initial} = \frac{1}{2}mv^2$

... next, $E_{k\,final}$:

formula: $\frac{1}{2}mw^2$

$= \frac{1}{2}m(v^2\cos^2\theta + \frac{1}{9}v^2\sin^2\theta)$

... factor out v^2 :

$\therefore E_{k\,final} = \frac{1}{2}mv^2(\cos^2\theta + \frac{1}{9}\sin^2\theta)$

subbing into our equation :

$\frac{1}{2} \times \frac{1}{2}mv^2 = \frac{1}{2}mv^2(\cos^2\theta + \frac{1}{9}\sin^2\theta)$

$\frac{1}{2}\cancel{\frac{1}{4}mv^2} = \cancel{\frac{1}{2}mv^2}(\cos^2\theta + \frac{1}{9}\sin^2\theta)$

cancel $\frac{1}{2}mv^2$

$\frac{1}{2} = \cos^2\theta + \frac{1}{9}\sin^2\theta$

but need to get rid of $\sin^2\theta$ as only want equation in terms of $\cos\theta$

↳ use identity: $\sin^2\theta = 1 - \cos^2\theta$

$\frac{1}{2} = \cos^2\theta + \frac{1}{9}(1 - \cos^2\theta)$

$\frac{1}{2} = \cos^2\theta + \frac{1}{9} - \frac{1}{9}\cos^2\theta$

collect like terms

$\frac{7}{18} = \frac{8}{9}\cos^2\theta$

$\div 8/9$ $\quad \div 8/9$

$\Rightarrow \cos^2\theta = 7/16$

Question 5 continued

square root

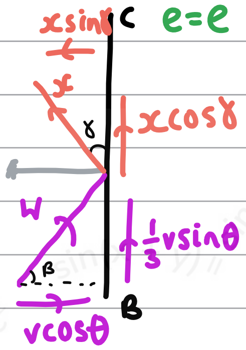
$\cos\theta = \pm\sqrt{7}/4$

but here take +ve

$\therefore \boxed{\sqrt{7}/4}$

(b) now let's focus on the second collision - the one with BC

SECOND COLLISION



... perpendicular:

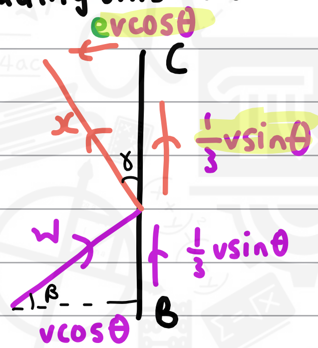
here, again, the IMPULSE acts perpendicular to the fixed wall BC, so only the perp. components change - here NEL rearranged applies:

$x\sin\gamma = ev\cos\theta$ ①

... parallel:

no change, so $x\cos\gamma = \frac{1}{3}v\sin\theta$ ②

↳ adding this to our diagram



and using the new information for E_k again - two main ways to do this:

METHOD 1: relating E_k final to initial collision with AB

i.e if the ball loses another half of its E_k in the second collision, then this implies that $\underbrace{\frac{1}{4}E_k \text{ initial}}_{\text{from AB}} = \underbrace{E_k \text{ final}}_{\text{from BC}}$

subbing into above:

$\frac{1}{4}\left(\frac{1}{2}mv^2\right) = \frac{1}{2}m\left(e^2v^2\cos^2\theta + \frac{1}{9}\sin^2\theta\right)$

factorise $\frac{1}{2}mv^2$ out on RHS

$\frac{1}{4}$ $\frac{1}{8}\cancel{mv^2} = \frac{1}{2}\cancel{mv^2}\left(e^2\cos^2\theta + \frac{1}{9}\sin^2\theta\right)$

cancel $\frac{1}{2}mv^2$

$\frac{1}{4} = e^2\cos^2\theta + \frac{1}{9}\sin^2\theta$

METHOD 2: relating E_k final to second collision with BC

if the ball loses $\frac{1}{2}$ of its energy remaining, then this implies

$\underbrace{\frac{1}{2}E_k \text{ initial}}_{\text{from BC}} = \underbrace{E_k \text{ final}}_{\text{from BC}}$

subbing into above:

$\frac{1}{2}\left(\frac{1}{2}m\left(v^2\cos^2\theta + \frac{1}{9}v^2\sin^2\theta\right)\right) =$

$\frac{1}{2}m\left(e^2v^2\cos^2\theta + \frac{1}{9}v^2\sin^2\theta\right)$

factorise $\frac{1}{2}mv^2$ out:

$\frac{1}{2}$ $\frac{1}{4}\cancel{mv^2}\left(\cos^2\theta + \frac{1}{9}\sin^2\theta\right) = \frac{1}{2}\cancel{mv^2}\left(e^2\cos^2\theta + \frac{1}{9}\sin^2\theta\right)$

Question 5 continued

using the identity: $\sin^2\theta = 1-\cos^2\theta$ to get rid of the $\sin^2\theta$

$\frac{1}{4} = e^2\cos^2\theta + \frac{1}{9}(1-\cos^2\theta)$

expand brackets

$\frac{1}{4} = e^2\cos^2\theta + \frac{1}{9} - \frac{1}{9}\cos^2\theta$

collect like terms :

$\frac{5}{36} = e^2\cos^2\theta - \frac{1}{9}\cos^2\theta$

sub in $\cos\theta = \frac{\sqrt{7}}{4}$ to above:

$\frac{5}{36} = e^2\left(\frac{7}{16}\right) - \frac{1}{9}\left(\frac{7}{16}\right)$

$\Rightarrow \frac{7}{16}e^2 = \frac{3}{16}$

$\div 7/16$ $\div 7/16$

$e^2 = 3/7$

square root both sides

$e = \pm\sqrt{3/7}$

but $0 \le e \le 1$

so $\boxed{e = \sqrt{3/7}}$

cancel $\frac{1}{2}mv^2$ and expand scalar

$\frac{1}{2}\cos^2\theta + \frac{1}{18}\sin^2\theta = e^2\cos^2\theta + \frac{1}{9}\sin^2\theta$

collect like terms

$\frac{1}{2}\cos^2\theta - e^2\cos^2\theta = \frac{1}{18}\sin^2\theta$

using identity $\sin^2\theta = 1-\cos^2\theta$

$\frac{1}{2}\cos^2\theta - e^2\cos^2\theta = \frac{1}{18}(1-\cos^2\theta)$

expand brackets

$\frac{1}{2}\cos^2\theta - e^2\cos^2\theta = \frac{1}{18} - \frac{1}{18}\cos^2\theta$

collect like terms

$\frac{5}{9}\cos^2\theta - e^2\cos^2\theta = \frac{1}{18}$

sub in $\cos\theta = \sqrt{7}/4$

$\frac{5}{9}\left(\frac{7}{16}\right) - e^2\left(\frac{7}{16}\right) = \frac{1}{18}$

expand brackets

$\frac{35}{144} - \frac{7}{16}e^2 = \frac{1}{18}$

$\frac{7}{16}e^2 = \frac{3}{16}$

$\div 7/16$ $\div 7/16$

$e^2 = 3/7$

square root both sides

$e = \pm\sqrt{3/7}$

but $0 < e < 1$, so

$\boxed{e = \sqrt{3/7}}$

(Total for Question 5 is 10 marks)

6.

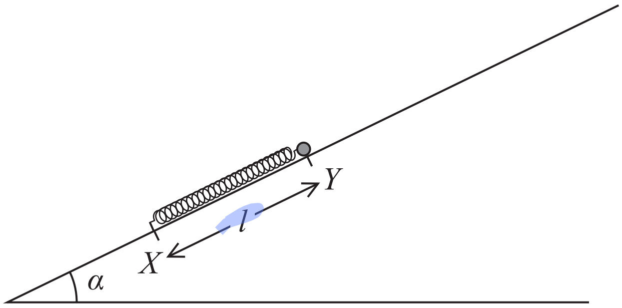


Figure 2

A light elastic spring has natural length $3l$ and modulus of elasticity $3mg$.

One end of the spring is attached to a fixed point X on a rough inclined plane.

The other end of the spring is attached to a package P of mass m.

The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$

The package is initially held at the point Y on the plane, where $XY = l$. The point Y is higher than X and XY is a line of greatest slope of the plane, as shown in Figure 2.

The package is released from rest at Y and moves up the plane.

The coefficient of friction between P and the plane is $\frac{1}{3}$

By modelling P as a particle,

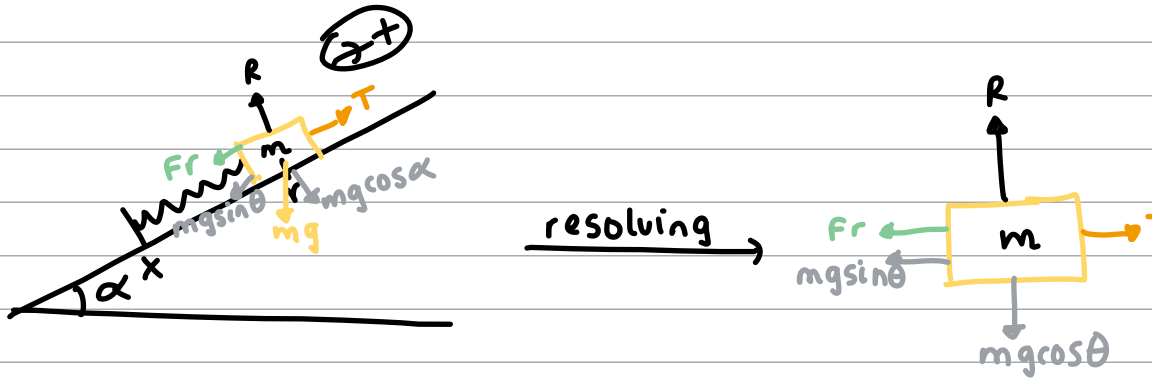
(a) show that the acceleration of P at the instant when P is released from rest is $\frac{17}{15}g$

(5)

(b) find, in terms of g and l, the speed of P at the instant when the spring first reaches its natural length of $3l$.

(6)

(a) considering this a 'dynamics' question, so have to include a detailed force diagram - includes thrust in spring ('compressed'), weight, resistance to motion (from 'rough, inclined plane')



P 6 6 8 0 0 A 0 2 0 2 8

Question 6 continued

know from Yr 2 Mechanics Chp 2 to use Newton's Second law for acceleration, hence resolve parallel (up) the plane

formula: $\Sigma F = ma$

$R(\nearrow): T - Fr - mg\sin\theta = ma$

↳ thrust formula: $T = \frac{\lambda x}{l}$ (elasticity, extension, natural length)

$= \frac{3mg(3l - l)}{3l}$

$= \frac{\cancel{3}mg(2\cancel{l})}{\cancel{3}\cancel{l}}$

$= 2mg$

↳ friction formula: $Fr = \mu R$ (coefficient of restitution, reaction force)

have to resolve perp. to plane
$R(\nwarrow): R = mg\cos\theta$

$= \frac{1}{3}(mg\cos\theta)$

$\therefore Fr = \frac{1}{3}mg\cos\theta$

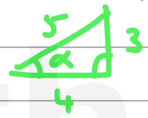
sub into equation

$2\cancel{m}g - \frac{1}{3}\cancel{m}g\cos\theta - \cancel{m}g\sin\theta = \cancel{m}a$

cancel m's

$2g - \frac{1}{3}g\cos\theta - g\sin\theta = a$

finally use fact that $\tan\alpha = \frac{3}{4}$
to form a trig triangle (3-4-5 Pythag. triple)



$\sin\alpha = \frac{O}{H} = \frac{3}{5}$

$\cos\alpha = \frac{A}{H} = \frac{4}{5}$

Sub in:

$2g - \frac{1}{3}g(4/5) - g(3/5) = a$

expand and collect like terms

$\Rightarrow \boxed{a = 17/15\, g \ (ms^{-2})}$

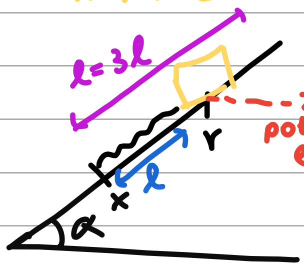
(b) the fact that we're asked to find the speed implies we're using E_k and hence the conservation of mechanical energy formula

Question 6 continued

let's draw the diagram for the INITIAL POSITION of the small ball at point Y, then next to it the position at which the small ball reaches its natural length

↳ label each with appropriate energies

INITIAL

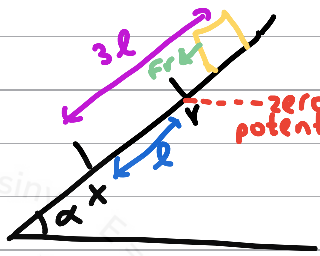


$\lambda = 3mg$

$x = 3l - l$

$= 2l$

FINAL



- E.P.E (compressed)

- K.E (need 'speed')
- G.P.E - need perp. distance (travelled $2l$)

using TRIG - see $h = 2l \sin\alpha$

from (a), $\sin\alpha = 3/5$

so $h = 6/5\, l$

- w.d by Fr : $Fr \times d$

$= \frac{4}{15} mg (2l)$

$= 8/15\, mgl$

now sub all into work-energy principle (includes dissipative forces)

$$\underset{n/a}{\text{w.d in}} + \underset{\text{initial kinetic}}{K.E_i} + \underset{\text{initial gravitational potential}}{G.P.E_i} + \underset{\text{initial elastic potential}}{E.P.E_i} = \underset{\text{final kinetic energy}}{K.E_f} + \underset{\text{final gravitational potential}}{G.P.E_f} + \underset{\text{final elastic potential}}{E.P.E_f} + \text{w.d against friction}$$

formula: $\frac{1}{2}mu^2 + mgh_1 + \frac{\lambda x^2}{2l} = \frac{1}{2}mv^2 + mgh_2 + \frac{\lambda x^2}{2l} + Fr \times d$

subbing into above:

$$0 + 0 + \frac{3mg(2l)^2}{2(3l)} = \frac{1}{2}mv^2 + \frac{6}{5}mgl + 0 + \frac{8}{15}mgl$$

simplify

$$2\cancel{m}gl = \frac{1}{2}\cancel{m}v^2 + \frac{6}{5}\cancel{m}gl + \frac{8}{15}\cancel{m}gl$$

cancel m's and collect like terms

DO NOT WRITE IN THIS AREA

Question 6 continued

$$\frac{1}{2}v^2 = \frac{4}{15}gl$$

$\div \frac{1}{2}$ $\div \frac{1}{2}$

$$v^2 = \frac{8}{15}gl$$

square root both sides

$$\boxed{v = \sqrt{\frac{8gl}{15}}\ (\mathrm{ms}^{-1})}$$

(Total for Question 6 is 11 marks)

7. [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

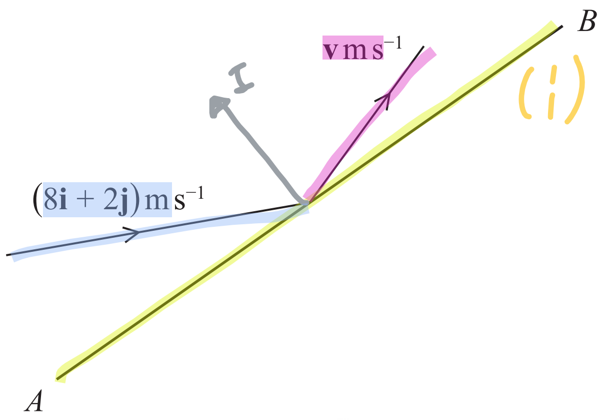


Figure 3

Figure 3 represents the plan view of part of a smooth horizontal floor, where *AB* is a fixed smooth vertical wall.

The direction of $\overrightarrow{AB}$ is in the direction of the vector $(\mathbf{i} + \mathbf{j})$

A small ball of mass 0.25 kg is moving on the floor when it strikes the wall *AB*.

Immediately before its impact with the wall *AB*, the velocity of the ball is $(8\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$

Immediately after its impact with the wall *AB*, the velocity of the ball is $\mathbf{v}\,\mathrm{m\,s^{-1}}$

The coefficient of restitution between the ball and the wall is $\frac{1}{3}$

By modelling the ball as a particle,

(a) show that $\mathbf{v} = 4\mathbf{i} + 6\mathbf{j}$

(6)

(b) Find the magnitude of the impulse received by the ball in the impact.

(3)

(a) recognising this as an oblique collision in 2D question but no fixed vertical wall - hence need to use our two key formulae for this type of collision

formula: $u \cdot w = v \cdot w$
initial velocity — vector wall — final velocity — vector wall

formula: $-eu \cdot I = v \cdot I$
initial speed — impulse — coefficient of restitution — final velocity — impulse

let $\underline{v} = \begin{pmatrix} a \\ b \end{pmatrix}$

..first trying the first formula:

P 6 6 8 0 0 A 0 2 4 2 8

Question 7 continued

$$\begin{pmatrix}8\\2\end{pmatrix}\cdot\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}a\\b\end{pmatrix}\cdot\begin{pmatrix}1\\1\end{pmatrix}$$

$\Rightarrow 10 = a+b$ ①

...next, the second formula, but for this need the vector perpendicular to the given wall vector - this vector will be our impulse.

...couple of ways to find the perp. vector to $\begin{pmatrix}1\\1\end{pmatrix}$:

WAY 1: using dot product and inspection

know that I needs to be perpendicular to $\begin{pmatrix}1\\1\end{pmatrix}$ so

$\underline{I}\cdot\begin{pmatrix}1\\1\end{pmatrix}=0$

$\therefore$ by inspection,

$\underline{I}=\begin{pmatrix}-1\\1\end{pmatrix}$ or $\begin{pmatrix}1\\-1\end{pmatrix}$

WAY 2: turning vectors into linear equations and exploit perp. lines properties

$\underline{w}=\begin{pmatrix}1\\1\end{pmatrix}$ - could interpret as gradient of line: $y=x$

$\Rightarrow m=1$

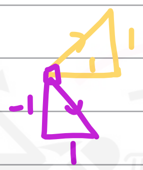
so using $m_1\times m_2=-1$, the line perp.,

$m=-1$

$\therefore$ line: $y=-x$,

$\therefore$ impulse: $\begin{pmatrix}-1\\1\end{pmatrix}$ or $\begin{pmatrix}1\\-1\end{pmatrix}$

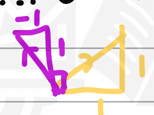
WAY 3: think of it as a transformation question - rotation



} 90° rotation clockwise

$\therefore I=\begin{pmatrix}1\\-1\end{pmatrix}$

... or anticlockwise



$\therefore I=\begin{pmatrix}-1\\1\end{pmatrix}$

so can use either $I=\begin{pmatrix}1\\-1\end{pmatrix}$ or $\begin{pmatrix}-1\\1\end{pmatrix}$

... eg. with $\begin{pmatrix}1\\-1\end{pmatrix}$, sub into second formula

$$-\frac{1}{3}\begin{pmatrix}8\\2\end{pmatrix}\cdot\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}a\\b\end{pmatrix}\cdot\begin{pmatrix}1\\-1\end{pmatrix}$$

evaluate scalar product

$-\frac{1}{3}(6)=a-b$

$\Rightarrow a-b=-2$ ②

Solve ① and ② simultaneously

① − ②

$$\begin{aligned} a+b&=10\\ -\quad a-b&=-2\\ \hline 2b&=12 \quad \div 2\\ b&=6\end{aligned}$$

Question 7 continued

sub $b=6$ into any of ① or ② for 'a'

eg. into ① : $6+b=10$

$\Rightarrow b=4$

$\boxed{\therefore \underline{v} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}}$ as required

(b) now can find the exact Impulse through subbing into the Impulse-momentum formula:

formula $I = m(\underline{v}-\underline{u})$

$I = 0.25\left(\begin{pmatrix} 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 8 \\ 2 \end{pmatrix}\right)$

$= 0.25\begin{pmatrix} -4 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\therefore$ finding magnitude using Pythagoras':

$I = \sqrt{(-1)^2 + (1)^2}$

$\boxed{= \sqrt{2} \text{ Ns}}$

NOTE : could've used the perp. vector logic from (a) and still get the same magnitude, but always good to double check with the formula (method marks!)

Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS